Robust methods for detecting hidden periodicity in models with additive non-Gaussian noise

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Introduction I

- We tackle the challenge of identifying hidden periodicity in signals that display periodic correlation while being influenced by non-Gaussian noise.
- This situation arises frequently across various fields.
- Traditional methods for detecting periodically correlated (PC) behavior typically rely on analyses in either the time domain or the frequency domain.
- In our study, we adopt these methods as well but introduce robust alternatives to the classical estimators for the autocovariance function and the discrete Fourier transform.
- Building on these techniques, we develop robust versions of widely used statistical methods initially designed to detect hidden periodicity in pure PC models.

Introduction II

- In our research we examined two types PC models and two types of non-Gaussian additive noise.
- Detecting hidden periodicity under non-Gaussian noise is considerably more complex than in standard cases.
- Using Monte Carlo simulations, we validate the proposed robust methods, demonstrating their effectiveness and superiority over traditional approaches.
- To further support our conclusions, we analyze real datasets where hidden periodicity has been previously confirmed in the literature.
- The vibration-based condition monitoring serves a key inspiration for our research.

Periodically correlated time series with additive noise I

We consider the following model, called periodically correlated model with additive noise

$$Y_t = X_t + Z_t, \;\; t \in \mathbb{Z},$$

where $\{X_t\}$ is a periodically correlated (PC) time series (random sequence) with period $T \in \mathbb{N}^*$, and $\{Z_t\}$ is the additive noise (AN). We assume that $\{Z_t\}$ is a stationary time series with a non-Gaussian distribution that is independent of $\{X_t\}$.
The time series $\{X_t\}$ is periodically correlated (or second-order cyclostationary) if its mean and autocovariance functions are periodic in t with the same period T

$$\mathbb{E}X_t = \mathbb{E}X_{t+T}, \qquad cov(X_t, X_{t+h}) = cov(X_{t+T}, X_{t+h+T}), \;\; h \in \mathbb{Z}.$$

Periodically correlated time series with additive noise II

- **PC model 1**

$$X_t = s(t) + \xi_t,$$

where $\{\xi_t\}$ is the sequence of Gaussian $\mathcal{N}(0,1)$ independent identically distributed (i.i.d.) random variables. The function $s(t)$ is a periodic function.

- **PC model 2** (PAR(p) model)

$$X_t - \phi_1(t)X_{t-1} - \dots - \phi_p(t)X_{t-p} = \xi_t,$$

where the $\{\xi_t\}$ is a sequence of i.i.d. random variables from $\mathcal{N}(0,1)$ distribution. The parameter sequences $\{\phi_i(t),\ i = 1, \dots, p\}$ are periodic with the same period $T \in \mathbb{N}^*$ with respect to t.

Periodically correlated time series with additive noise III

- **AN model 1**

$$Z_t = A_t K_t, \quad A_t \in U(0, a).$$

$\{K_t\}$ is an i.i.d. sequence of the following distribution

$$\mathbb{P}(K_t = 1) = \mathbb{P}(K_t = -1) = q/2, \quad \mathbb{P}(K_t = 0) = 1 - q.$$

- **AN model 2**

$$Z_t - \tilde{\phi}_1 Z_{t-1} - \ldots - \tilde{\phi}_p Z_{t-p} = \xi_t,$$

where the $\{\xi_t\}$ is a sequence of i.i.d. random variables with symmetric $\alpha-$stable distribution defined via the characteristic function

$$\Phi(z) = \mathbb{E}\exp\{i\xi_t z\} = \exp\left(-\sigma^\alpha |z|^\alpha\right), \qquad z \in \mathbb{R},$$

where $\alpha \in (0, 2]$ – stability index, $\sigma > 0$ – scale parameter.

Periodically correlated time series with additive noise IV

We consider the following setups of presented PC models (with period $T = 8$) and additive noise:

- PC model 1 (later denoted as PC1): $s(t) = 1 + \sin\left(\frac{1}{4}\pi t\right)$, $\xi_t \sim \mathcal{N}(0, 1)$,
- PC model 2 (PC2): PAR(1) model with $\phi_1(1) = -0.6$, $\phi_1(2) = 1.7$, $\phi_1(3) = 0.9$, $\phi_1(4) = -0.4$, $\phi_1(5) = 0.8$, $\phi_1(6) = -0.8$, $\phi_1(7) = 0.7$, $\phi_1(8) = -0.2$, $\xi_t \sim \mathcal{N}(0, 1)$,
- AN model 1 (AN1): $a = 60$, $q = 0.005$,
- AN model 2 (AN2): AR(1) model with $\tilde{\phi}_1 = 0.2$, $\alpha = 1.8$, $\sigma = 1$.

Periodically correlated time series with additive noise V

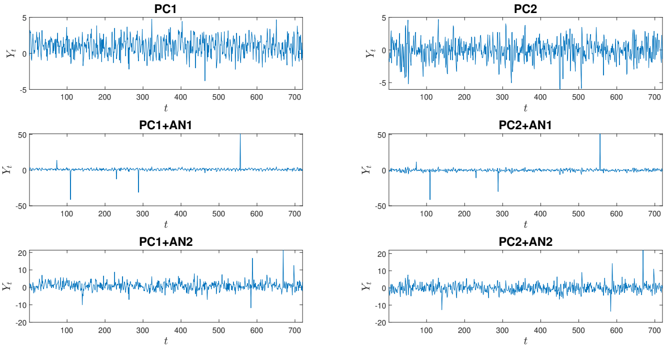


Figure: Sample trajectories of PC1 (left column) and PC2 (right column) time series: without additive noise (top row), and with additive noise from AN1 (middle row) and AN2 (bottom row) models.

Robust coherent and incoherent statistics in application to local damage detection problem

Introduction

- The coherent and incoherent statistics are classical tools for the detection of periodic behaviour, which is a key part of many local damage detection procedures.
- However, standard methods tend to fail if the signal of interest is disturbed by heavy-tailed non-Gaussian noise (which might be a case in real data).
- Hence, we propose robust versions of coherent and incoherent statistics (based on the M-Fourier transform).
- Moreover, we consider their application to signals represented in time-frequency domain (spectrograms) to create robust coherent/incoherent statistic maps.
- The presented methodology is applied to simulated and real signals.

Coherent/incoherent statistics [1, 2]

For signal $\mathbf{X} = [X_1, \ldots, X_N]$, the sample coherence is defined as:

$$|\gamma(p,q,K)|^2 = \frac{|\sum_{k=0}^{K-1} I(\omega_{p+k})\overline{I(\omega_{q+k})}|^2}{\sum_{k=0}^{K-1} |I(\omega_{p+k})|^2 \sum_{k=0}^{K-1} |I(\omega_{q+k})|^2}, \quad 0 < p, q \leq N,$$

where $I(\omega_1), \ldots, I(\omega_N)$ is the discrete Fourier transform from $\mathbf{X}$, and K is the smoothness coefficient (hyperparameter).

Using the sample coherence, let us define for $0 < d < N$:

- coherent statistic:

$$\delta_C(d) = |\gamma(0, d, N)|^2$$

- incoherent statistic (for specified K):

$$\delta_I(d) = \frac{1}{L+1} \sum_{p=0}^{L} |\gamma(pK, pK+d, K)|^2 \quad (L = [(N-1-d)/K])$$

Robust coherent/incoherent statistics I

- The coherent/incoherent statistics are sensitive to outliers.
- Idea: replace the Fourier transform with its robust version.
- Here, we use the modification based on the M-regression [3].

Fourier transform as a linear regression problem:

$$I(\omega_j) = N/2(\hat{\beta}_{1,j} - i\hat{\beta}_{2,j}) \quad j = 1, \ldots, N,$$

where the coefficients $\hat{\beta}_j = [\hat{\beta}_{1,j}, \hat{\beta}_{2,j}]'$ are

$$\hat{\beta}_j = \underset{\beta_j \in \mathbb{R}^2}{\operatorname{argmin}} \left[\sum_{r=0}^{N-1} \left(X_r - \mathbf{C}_r' \beta_j \right)^2 \right]$$

for $\mathbf{C}_r = [\cos(2\pi r \omega_j), \sin(2\pi r \omega_j)]'$.

Robust coherent/incoherent statistics II

- Key step: replace the least squares cost function with a more robust one (here: Huber function)

$$\rho(x) = \begin{cases} x^2/2, & |x| \leq c \\ c(|x| - c/2), & |x| > c \end{cases}, \quad c - \text{ tuning constant}$$

To obtain a more robust version of the Fourier transform, we set:

$$\hat{\beta}_j = \underset{\beta_j \in \mathbb{R}^2}{\operatorname{argmin}} \left[\sum_{r=0}^{N-1} \rho\left((X_r - \mathbf{C}_r' \beta_j)/s \right) \right],$$

and use obtained values to calculate $I(\omega_j)$.

In robust coherent/incoherent statistics, we use the above robust version of $I(\omega_j)$ in the sample coherence formula.

Robust coherent/incoherent statistics III

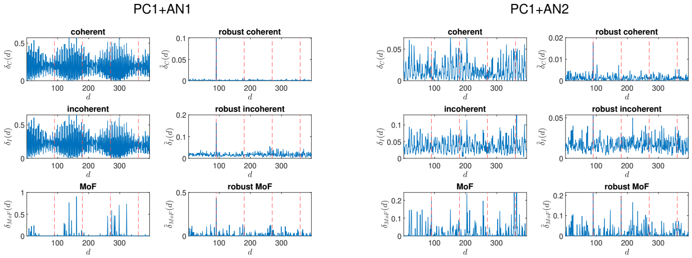


Figure: Analyzed periodicity detection statistics calculated for PC1+AN1 and PC1+AN2 trajectories. The cyclic $d \in D = \{90, 180, 270, 360\}$ are marked with red dashed lines.

Robust coherent/incoherent statistics IV

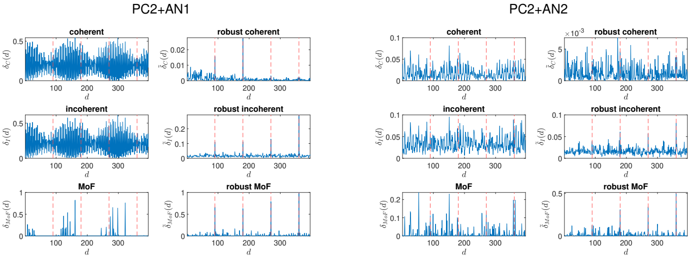


Figure: Analyzed periodicity detection statistics calculated for PC2+AN1 and PC2+AN2 trajectories. The cyclic $d \in D = \{90, 180, 270, 360\}$ are marked with red dashed lines.

Robust coh./incoh. statistics for signals in t-f domain

To construct bi-frequency maps for the periodicity detection using robust coh./incoh. statistics, we use the following algorithm:

- construct the spectrogram $S(t, f)$: time-frequency representation of the data

  $$S(t, f) = |\mathrm{STFT}(t, f)|^2,$$

  where $\mathrm{STFT}(t, f)$ is the short-time Fourier transform of given signal $\mathrm{STFT}(t, f) = \sum_{n=0}^{N-1} X_n w(t-n) \exp(-2\pi i f n/N)$, with $w(t-n)$ is a shifted window of length N_w.
- for each f:
  - take $\mathbf{S}_f = [S(t_1, f), \ldots, S(t_N, f)]$: spectrogram row for f
  - from $\mathbf{S}_f$, for $\epsilon_{min} \leq \epsilon \leq \epsilon_{max}$ (modulation frequency), calculate $\delta_C(d)$ (coherent) or $\delta_I(d)$ (incoherent), setting $d = \epsilon t_N$.

Application to simulated and real signals I

Signal 1: signal simulated from the model **PC 1+AN 2**

$$X_t = s(t) + Z_t,$$

where $s(t)$ - cyclic impulses, called the signal of interest (SOI). We assume a specific form of $s(t)$, which is composed of a series of individual impulses located in time with a given period $T = 1/f_f$, where f_f is a fault frequency. A single impulse may be specified as a decaying harmonic oscillation of the following form

$$h(t) = B \sin(2\pi f_c t) e^{-dt}, \quad t \geq 0$$

B is the amplitude, f_c is the carrier frequency (related to the structural resonance in the machine). We assume: fault frequency $f_f = 30$ Hz, amplitude $B = 45$, informative frequency band $f_c = 3500 - 6500$ Hz.
$\{Z_t\}$ – sequence of i.i.d. random variables from symmetric α-stable distribution (with $\alpha = 1.7, \sigma = 3$).

Application to simulated and real signals II

Signal 2: real vibration signal from a (healthy) crushing machine with added cyclic impulses $s(t)$ (of amplitude $B = 0.25$).
Both signals consist of $L = 50000$ observations (sampling frequency 25000 Hz, 2 seconds).

Figure: The exemplary crushing machine in the copper ore mine.

Application to simulated and real signals III

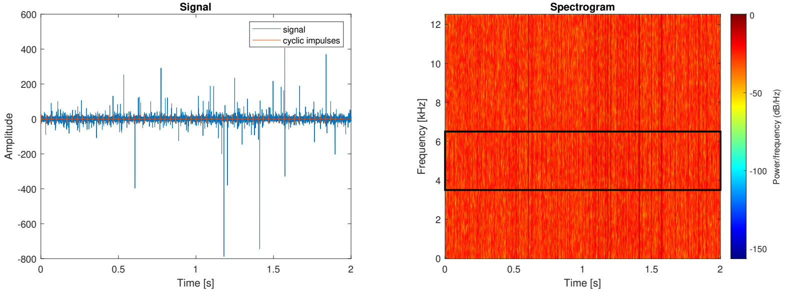


Figure: Signal 1 and its spectrogram.

Application to simulated and real signals IV

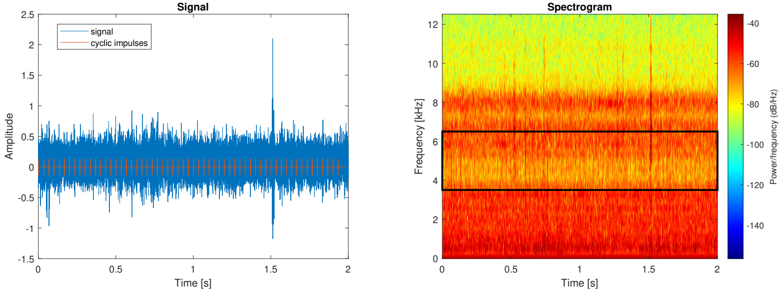
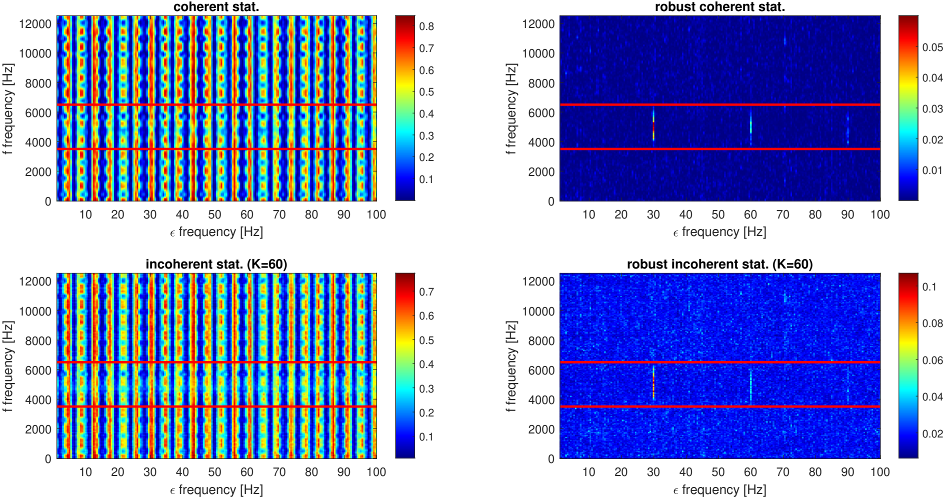


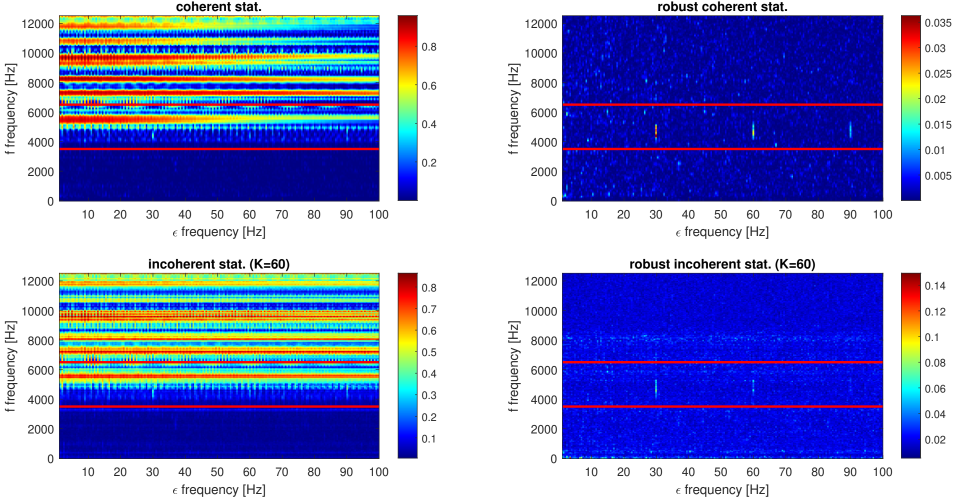
Figure: Signal 2 and its spectrogram.

Application to simulated and real signals V



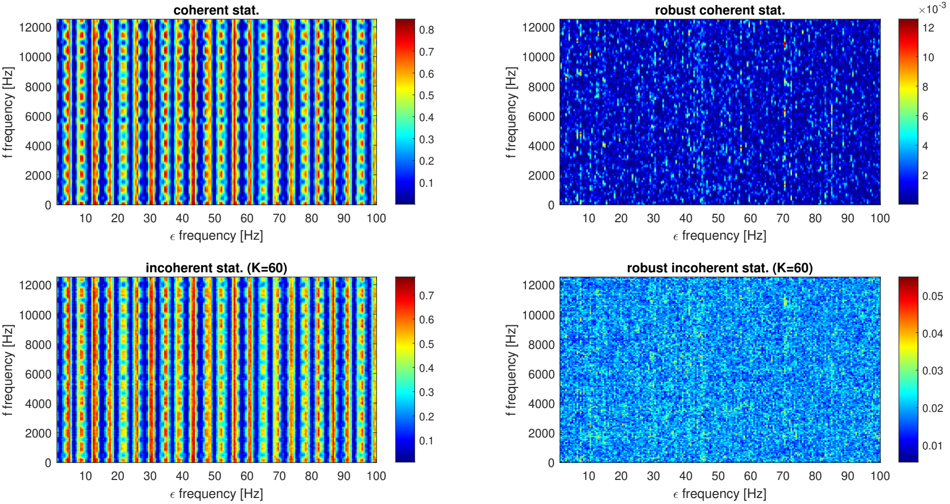
Maps obtained for Signal 1.

Application to simulated and real signals VI



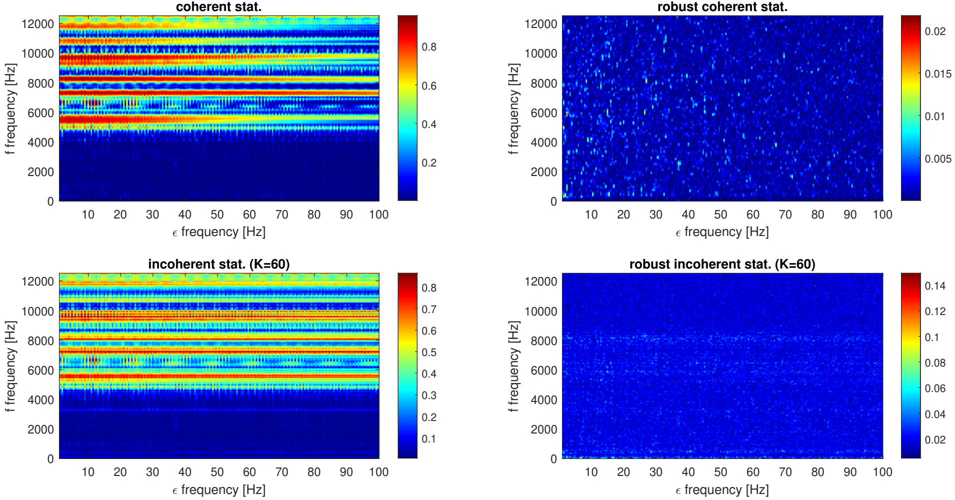
Maps obtained for Signal 2.

Application to non-periodic signals I



Maps obtained for Signal 1 without periodicity.

Application to non-periodic signals II



Maps obtained for Signal 2 without periodicity.

Robust estimators of autocorrelation function in application to local damage detection problem

Introduction

- Many local damage detection procedures are based on the periodicity detection methods which utilize the autocorrelation function (ACF) and its standard estimator (sample ACF).
- However, classical methods may fail if the signal of interest is disturbed by heavy-tailed non-Gaussian noise.
- Hence, we propose to use robust ACF estimators in periodicity detection algorithms.
- Here, we consider robust modification of the spectral coherence map [4].
- The presented methodology is applied to simulated and real signals.

Cyclic spectral coherence I

Frequency-frequency domain analysis – spectral coherence map $|\gamma(f,\epsilon)|^2$ (ϵ – cycle frequency) [4, 6, 7]

Definition (Cyclic spectral coherence)

For a finite-variance process $\{X_t\}$, cyclic spectral coherence is defined as follows

$$\gamma(f,\epsilon) = \frac{|S_X(f,\epsilon)|^2}{S_X(f+\epsilon/2,0)S_X(f-\epsilon/2,0)},$$

where

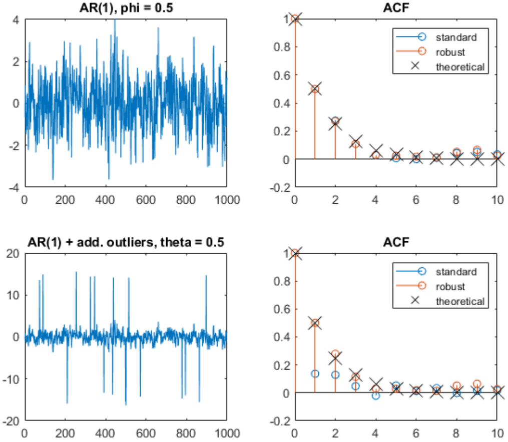
$$S_X(f,\epsilon) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=-N}^{N} \sum_{\tau=-\infty}^{\infty} R_X(t,\tau) e^{-i2\pi f\tau} e^{-i2\pi\epsilon t}$$

for $R_X(t,\tau) = \mathbb{E}X_t X_{t-\tau}$ being the autocovariance function of process $\{X_t\}$, and ϵ is the cycle frequency.

Cyclic spectral coherence II

In practice, there are several ways to estimate the spectral coherence, e.g. the averaged cyclic periodogram (ACP) method. As a result, we obtain a bi-frequency map. Classical CSC map is constructed with methods that use the sample ACVF which is very sensitive to outliers.

Example: ACF in presence of outliers



Example of standard and robust ACF estimation for AR(1) sample without (top) and with additive outliers (bottom).

Robust ACF estimators [5]

Estimation of (Pearson) correlation between centered* vectors $\mathbf{w}_1 = (w_1^1, w_1^2, \cdots, w_1^N)$ and $\mathbf{w}_2 = (w_2^1, w_2^2, \cdots, w_2^N)$:

- (non-robust) sample ACF (numerator: sample ACVF)

$$\boxed{M_1(\mathbf{w}_1, \mathbf{w}_2) = \frac{\frac{1}{N}\sum_{i=1}^N w_1^i w_2^i}{\frac{1}{N}\sqrt{\sum_{i=1}^N (w_1^i)^2 \sum_{i=1}^N (w_2^i)^2}}}$$

- trimmed estimator with parameter $0 \leq c < 0.5$ (trimm)

$$\mathbf{w}_3 = (w_3^1, w_3^2, \cdots, w_3^N) = (w_1^1 w_2^1, w_1^2 w_2^2, \cdots, w_1^N w_2^N)$$

$$\tilde{\mathbf{w}}_k = \{w_k^i \,:\, i = 1, \ldots, N;\ w_3^{(g)} < w_3^i < w_3^{(n-g+1)}\}, \quad k = 1, 2$$

where $(w_3^{(1)}, \ldots, w_3^{(N)})$ – ordered $\mathbf{w}_3$, $g = \lfloor c \cdot N \rfloor$

$$\boxed{M_2^c(\mathbf{w}_1, \mathbf{w}_2) = M_1(\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2)}$$

*note: centering is usually done by subtracting the sample mean in non-robust and sample median in robust methods

Robust ACF estimators

- Kendall correlation:

$$\rho_K(\mathbf{w}_1, \mathbf{w}_2) = \frac{2}{N(N-1)} \sum_{1 \le i \le j \le N} \mathrm{sgn}((w_1^i - w_1^j)(w_2^i - w_2^j))$$

$$\boxed{M_3(\mathbf{w}_1, \mathbf{w}_2) = \sin\left(\frac{\pi \rho_K(\mathbf{w}_1, \mathbf{w}_2)}{2}\right)}$$

- Spearman correlation:
  ($\mathbf{r}_1$, $\mathbf{r}_2$ – zero-mean vectors of ranks for $\mathbf{w}_1$, $\mathbf{w}_2$)

$$\rho_S(\mathbf{w}_1, \mathbf{w}_2) = \frac{\sum_{i=1}^N r_1^i r_2^i}{\sqrt{\sum_{i=1}^N (r_1^i)^2 \sum_{i=1}^N (r_2^i)^2}} = M_1(\mathbf{r}_1, \mathbf{r}_2)$$

$$\boxed{M_4(\mathbf{w}_1, \mathbf{w}_2) = \sin\left(\frac{2\pi \rho_S(\mathbf{w}_1, \mathbf{w}_2)}{6}\right)}$$

α-stable distribution and covariation [6]

- Symmetric α-stable distribution $\mathcal{S}(\alpha, \sigma)$ has **infinite variance** for $\alpha < 2$ and reduces to **Gaussian distribution** for $\alpha = 2$.

- For two random variables S_1, S_2 from symm. α-stable distribution ($\alpha > 1$), we define the normalized covariation of S_1 on S_2 as

$$NCV(S_1, S_2) = \frac{\mathbb{E}(S_1 \operatorname{sgn}(S_2))}{\mathbb{E}|S_2|}.$$

  Estimation of NCV of centered vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ (sample NCV):

$$\lambda(\mathbf{w}_1, \mathbf{w}_2) = \frac{\sum_{i=1}^{N} w_1^i \operatorname{sgn}(w_2^i)}{\sum_{i=1}^{N} |w_2^i|}.$$

Robust spectral coherence maps I

Algorithm 2 Robust spectral coherence for a signal $X = x_1, \dots, x_L$.

- $\odot$ - element-wise multiplication of vectors
- $X[\text{index}] = [X_{\text{index}_1}, \dots, X_{\text{index}_n}]$, where $\text{index} = [\text{index}_1, \dots, \text{index}_n]$

Set $M(\cdot,\cdot)$ - selected robust covariance/correlation estimator
Set $w(\cdot)$ - window function of length n
Set nfft - number of sampling points to calculate DFT
Set nover - size of overlap
Set $\epsilon_{\min}$, $\epsilon_{\max}$ – minimal and maximal modulation frequency
$t = [0, 1, \dots, N-1]$
for $k \leftarrow \epsilon_{\min}$ to $\epsilon_{\max}$ **do**
    $K = \left\lfloor \frac{N-\text{nover}}{n-\text{nover}} \right\rfloor$
    $X^k = X \odot e^{i\pi k t}$
    $Y^k = X \odot e^{-i\pi k t}$
    $\text{index} = [1, \dots, n]$
    for $i \leftarrow 1$ to K **do**
        $X^w = w \odot X^k[\text{index}]$
        $Y^w = w \odot Y^k[\text{index}]$
        for $j \leftarrow 1$ to nfft **do**
            $X_w(j,i) = \text{DTFT}_{\text{nfft}}(j, X^w)$
            $Y_w(j,i) = \text{DTFT}_{\text{nfft}}(j, Y^w)$
        end for
        $\text{index} = \text{index} + (n - \text{nover})$
    end for
    for $j \leftarrow 1$ to nfft **do**
        $S_X(f_j, \epsilon_k) = M(Y_w(j,:), X_w(j,:)^*)$
    end for
    Calculate robust spectral coherence $|\gamma_X(f_j, \epsilon_k)|^2$
end for

Robust spectral coherence maps II

Calculation of robust ACF/ACVF estimators for complex inputs:

- For two complex-valued random variables $\xi_1 = \Re_1 + \Im_1 j$ and $\xi_2 = \Re_2 + \Im_2 j$ we have

$$\mathbb{E}\xi_1\xi_2^* = \mathbb{E}(\Re_1\Re_2) - \mathbb{E}(\Im_1\Im_2) + [\mathbb{E}(\Re_1\Im_2) + \mathbb{E}(\Im_1\Re_2)]j$$

  We obtain four real expectations where each can be estimated by the trimmed ACVF.
- In the Kendall method, we use $\mathrm{sgn}(z) = z/|z|$ for $z \in \mathbb{C}$
- For the Spearman estimator, we set

$$\rho_S(\mathbf{w}_1, \mathbf{w}_2) = M_1(\mathbf{r}_{\mathrm{Re}(\mathbf{w_1})} + \mathbf{r}_{\mathrm{Im}(\mathbf{w_1})}j, \mathbf{r}_{\mathrm{Re}(\mathbf{w_2})} + \mathbf{r}_{\mathrm{Im}(\mathbf{w_2})}j),$$

  where $\mathbf{r}_{\mathrm{Re}(\mathbf{w_i})}$ and $\mathbf{r}_{\mathrm{Im}(\mathbf{w_i})}$ are zero-mean vectors of ranks of respectively real and imaginary parts of vector $\mathbf{w}_i$, $i = 1, 2$.

Application to simulated and real signals I

- Signal 1: signal simulated from the model **PC 1+AN 2**

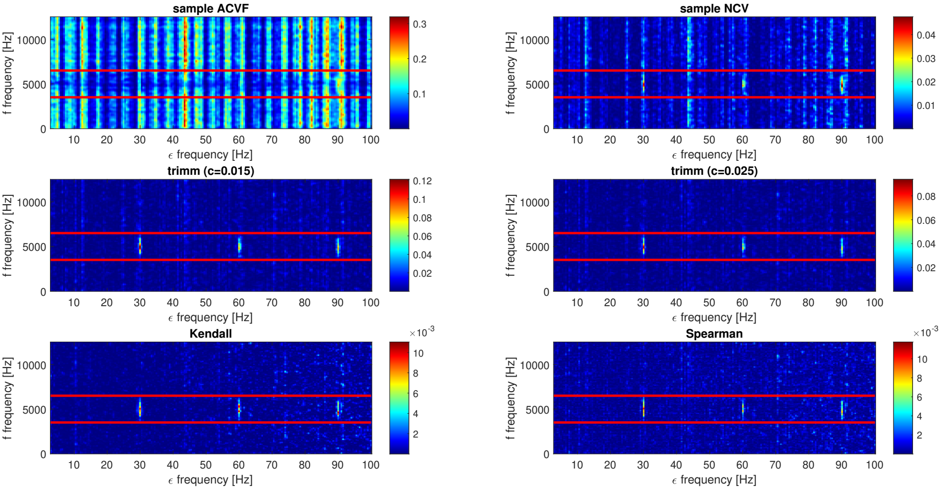
$$X_t = s(t) + Z_t$$

  - $s(t)$ - cyclic impulses
    - fault frequency $f_f = 30$ Hz, amplitude $B = 45$
    - informative frequency band $f_c = 3500 - 6500$ Hz
  - $\{Z_t\}$ – sequence of i.i.d. random variables from symmetric α-stable distribution (with $\alpha = 1.7, \sigma = 3$).

- Signal 2: real vibration signal from a (healthy) crushing machine with added cyclic impulses $s(t)$ (of amplitude $B = 0.25$).

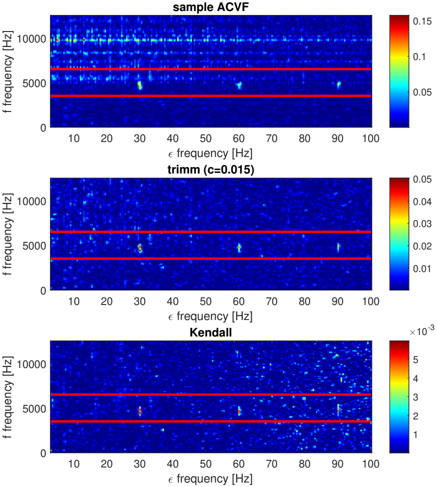
- Both signals consist of $L = 50000$ observations (sampling frequency 25000 Hz, 2 seconds).

Application to simulated and real signals II



Spectral coherence maps $|\gamma(f, \epsilon)|^2$ for Signal 1.

Application to simulated and real signals III



Spectral coherence maps $|\gamma(f,\epsilon)|^2$ for Signal 2.

Application to simulated and real signals IV

To compare the values of periodic impulses (in f_c band) with the noise on the map, we calculate amplitude ratio for each ϵ

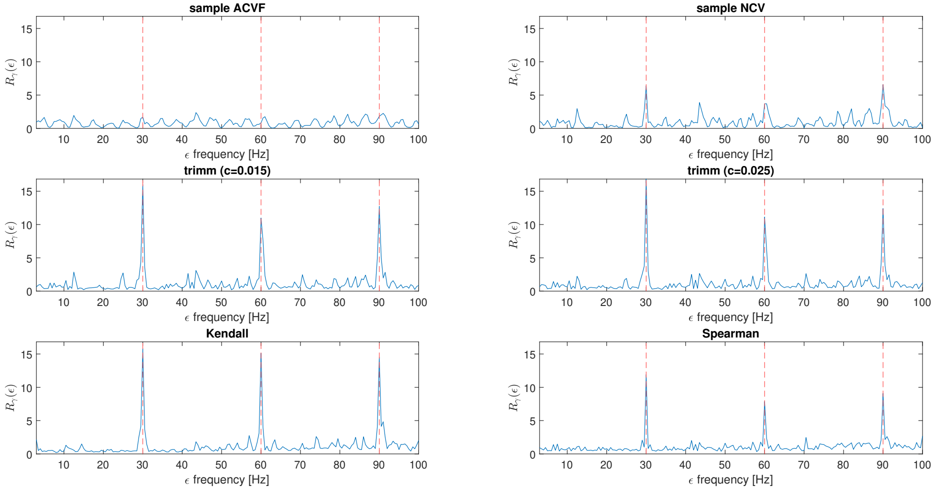
$$R_\gamma(\epsilon) = \frac{\overline{|\gamma(f_c, \epsilon)|^2}}{\overline{|\gamma|^2}},$$

where $\overline{|\gamma(f_c, \epsilon)|^2}$ is the mean of map values in f_c band for ϵ, and $\overline{|\gamma|^2}$ is the mean of all map values.

For an evaluation of the map, we consider the following indicator:

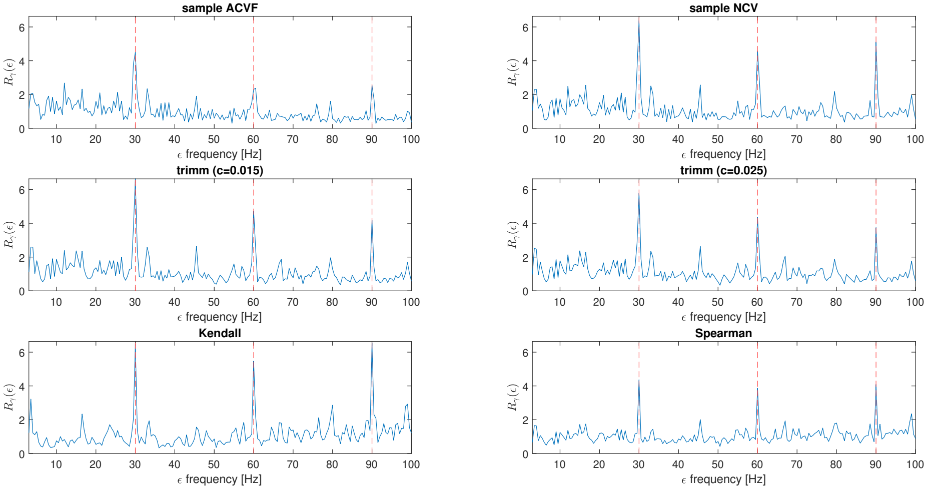
$$\tau_\gamma = \frac{\sum_{\epsilon \text{ cyclic}} R_\gamma(\epsilon)}{\sum_{\epsilon=\epsilon_{min}}^{\epsilon_{max}} R_\gamma(\epsilon)}$$

Application to simulated and real signals V



Amplitude ratios $R_\gamma(\epsilon)$ for Signal 1.

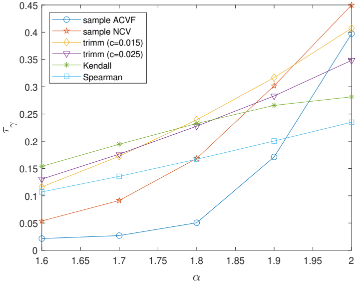
Application to simulated and real signals VI

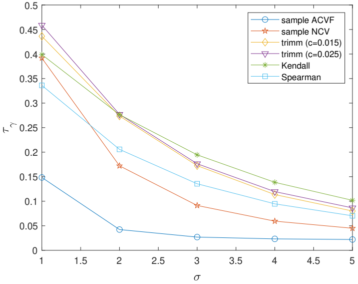


Amplitude ratios $R_\gamma(\epsilon)$ for Signal 2.

Application to simulated and real signals VII

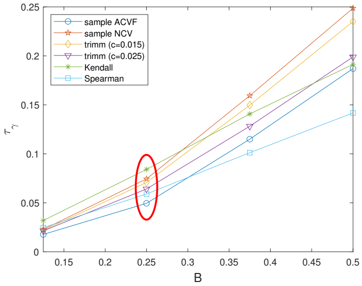
$$\{Z_t\} \sim \mathcal{S}(\alpha, \sigma = 3) \qquad \{Z_t\} \sim \mathcal{S}(\alpha = 1.7, \sigma)$$





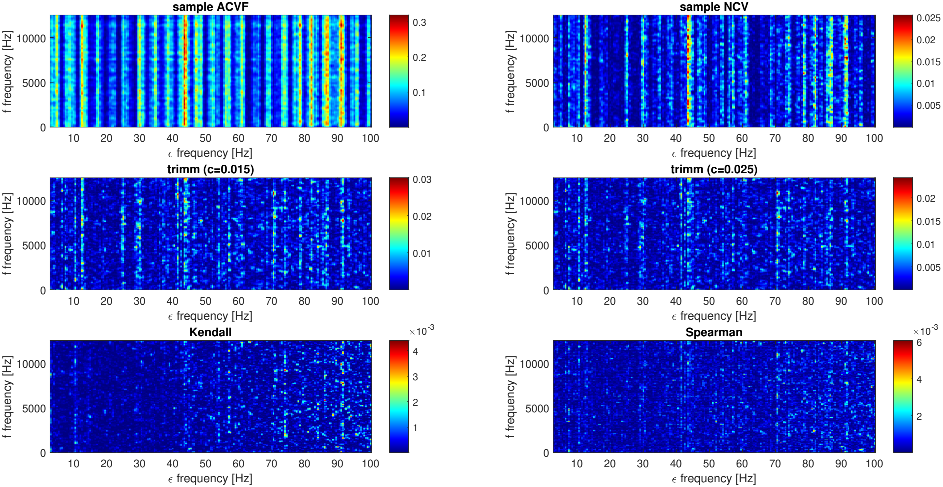
Values of τ_γ calculated for Signal 1 with different $\{Z_t\} \sim \mathcal{S}(\alpha, \sigma)$ cases.

Application to simulated and real signals VIII



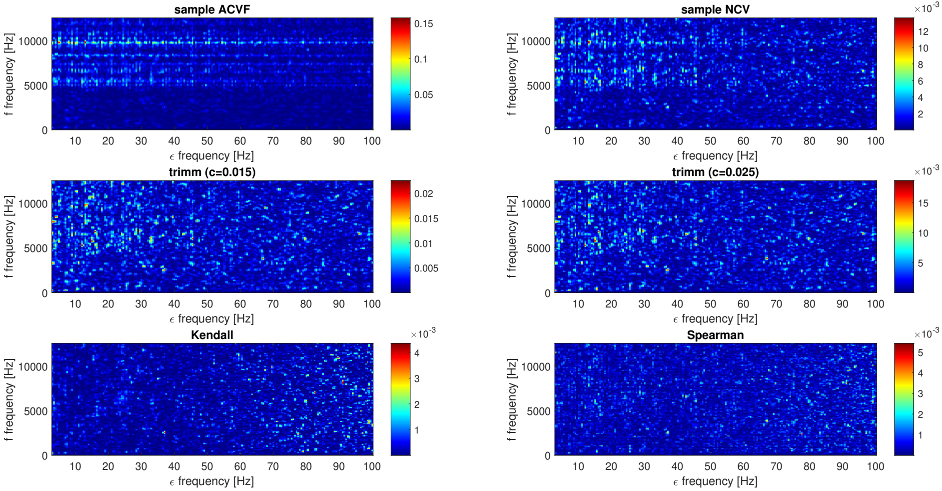
Values of τ_γ calculated for Signal 2 with different amplitudes B of added cyclic impulses.

Application to non-periodic signals I



Spectral coherence maps $|\gamma(f, \epsilon)|^2$ for Signal 1 without periodicity.

Application to non-periodic signals II



Spectral coherence maps $|\gamma(f,\epsilon)|^2$ for Signal 2 without periodicity.

Summary

- In this work, we presented the application of robust approaches for hidden periodicity detection when dealing with PC models with non-Gaussian additive noise.
- We present two approaches, (1) where the robust Fourier transform is used in the coherent/incoherent statistics calculation and (2) where the robust estimators of ACF are used in the cyclic spectral coherence map.
- The proposed approach outperfrmes classical (non-robust) methods when non-Gaussian behaviour is present in the analyzed signal.
- In practice, such behaviour may occur due to specific processes conducted by the machine (e.g. cutting, crushing, drilling).
- Our current research extends the methodology presented by the introduction of new health indices (HI) (based on robust maps) that allow tracking of progressing damage, [9].

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Thank you for your attention!